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B.Sc. SECOND SEMESTER EXAMINATION, MAY 2019
FIRST YEAR (BATCH 2018-21)
MATHEMATICS (General)
Paper : II

Date : 24/05/2019
Time : 11 am – 2 pm
Full Marks : 75

[Use a separate Answer Book for each group]

Group - A

Answer any three questions from Question No. 1 to 5 : [5×3=15]

1. a) Find the transformed equation of the straight line $\frac{x}{a}+\frac{y}{b}=2$ when the origin is transferred to the point (a,b).
   b) To what points must the origin be moved in order to remove the terms of first degree in the equation $2x^2-3y^2-4x-12y=0$. [2+3]

2. Reduce the following equation to its canonical form and determine the nature of the conic represented by : $3x^2+2xy+3y^2-16x+20=0$. [5]

3. Show that the equation to the pair of straight lines through origin perpendicular to the pair of straight lines $ax^2+2hxy+by^2=0$ is $bx^2-2hxy+ay^2=0$. [5]

4. Find the equation of the pair of tangents from an external point (x_1, y_1) to the parabola $y^2=4ax$. [5]

5. a) Deduce the polar equation of the circle, when the pole lies in the exterior of the circle.
   b) Find the centre of the circle, $r=3\sin\theta+4\cos\theta$. [3+2]

Answer any two questions from Question No. 6 to 8 : [5×2=10]

6. Reduce $x^2p^2+y(2x+y)p+y^2=0$ to Clairaut's form by the substitution y=u, xy=v. Hence find the general solution. [3+2]

7. Solve : $(y^2+2x^2y)dx+(2x^3-xy)dy=0$. [5]

8. Solve : $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$. [5]

Group – B

Answer any five questions from Question No. 9 to 16 : [5×5=25]

9. Find a and b in order that $\lim\limits_{x\to 0}\frac{x(a\cos x+1)-b\sin x}{x^3}$ is finite and equals to 1. [5]

10. a) Prove that the sequence $\{u_n\}$, defined by $u_1=\sqrt{2}$ and $u_{n+1}=\sqrt{2u_n}$ for all $n\geq 1$, converges to 2.

    b) If $\{x_ny_n\}$ is convergent, does it always mean that $\{x_n\}$ and $\{y_n\}$ both are convergent? [3+2]

11. State Rolle's theorem and use it to prove Lagrange's mean value theorem. [1+4]

12. Expand $\sin x$ in Maclaurin's infinite series. [5]

13. Find the maximum or minimum value of $x^2+y^2+z^2$ subject to $ax+by+cz=p$. [5]

14. Determine the asymptotes of the following curves :

a) $y=\dfrac{3x}{x-1}+3x$

b) $x^3-x^2y+ay^2=0$ [2+3]

15. Find the maxima and minima of $f(x)=1+2\sin x+3\cos^2 x$, $(0\le x\le \frac{\pi}{2})$. [5]

16. Apply Raabe's test to examine the convergence of $1+\frac{1}{2}\cdot\frac{1}{3}+\frac{1\cdot 3}{2\cdot 4}\cdot\frac{1}{5}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{1}{7}+\cdots\cdots$ [5]

Answer <u>any two</u> questions from <u>Question No. 17 to 19</u> : [5×2=10]

17. Integrate : $\int \dfrac{x}{(x-1)(x^2+4)}dx$. [5]

18. If $I_n=\int_0^{\pi/2} x^n \sin x\,dx$, (n>1, a positive integer) show that $I_n+n(n-1)I_{n-2}=n\left(\dfrac{\pi}{2}\right)^{n-1}$.

[5]

19. Prove that $\lim\limits_{n\to\infty}\left[\dfrac{1^2}{n^3+1^3}+\dfrac{2^2}{n^3+2^3}+\dfrac{3^2}{n^3+3^3}+\cdots\cdots+\dfrac{n^2}{2n^3}\right]=\dfrac{1}{3}\log_e 2$. [5]

Answer <u>any three</u> questions from <u>Question No. 20 to 24</u> : [5×3=15]

20. Prove that the necessary and sufficient condition for the collinearity three points A, B, C is that there exist scalars x, y, z, not all zero, such that $x\vec{a}+y\vec{b}+z\vec{c}=\vec{0}$, $x+y+z=0$, where $\vec{a},\vec{b},\vec{c}$ are respectively the position vectors of A, B, C with respect to some chosen origin. [3+2]

21. Find the work done by the force $\vec{F}=2\hat{i}+5j-k$, when it displaces a particle from the point A (3,6,9) to the point B (7,8,9). [5]

22. Show by vector method that $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$, where the symbols are of usual meaning. [5]

23. Find a unit vector in the plane of the vectors $(\hat{i}+2j-k)$ and $(\hat{i}+j-2k)$, which is perpendicular to the vector $(2\hat{i}-j+k)$. [5]

24. Find the vector equation of the plane through the origin and is parallel to the vectors $\hat{i}+2j+3k$ and $2\hat{i}-j-k$ in both parametric and non-parametric forms. [3+2]

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